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Tolerances for X-band structure

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- Introduction
- Random frequency errors
- Systematic frequency errors
- Compensation of systematic errors
- Conclusion

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Phase errors in structure and energy loss.

Introduction

In perfect accelerating structure the wave phase velocity is equal to beam velocity at designed frequency f_0. Manufactured errors in cell geometry causes errors in cell frequency and as a result, the wave phase velocity will differ from the beam velocity. Beam will accelerate out of wave crest, that decrease effective accelerating gradient. Cell with frequency error *df* gives additional phase shift *dq*:

$$dq(x) = \Theta_0 \cdot \frac{df}{f_0 \cdot b_g(x)}$$

Q_0=*2pD/l* - *phase advance per cell,*
b_g**c*– group velocity

$$q(x) = \int_0^x dq(x) \cdot dx = \frac{\Theta_0}{f_0} \cdot \int_0^x \frac{df(x)}{b_g(x)} \cdot dx$$

The integrated phase error along structure:
(x=z/L-coordinate along structure)

$$\frac{\Delta E}{E} = \left\langle \frac{\int (E(x) - E'(x) \cdot \cos(q(x)) \cdot dx}{\int E(x) \cdot dx} \right\rangle$$

Energy loss in LC

or

$$\frac{\Delta E}{E} = \int_0^1 (1 - \cos(q(x)) \cdot dx$$

For E(z)=const (NLC)

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Phase errors in structure and energy loss (2)

Cell frequency error: $$df = \sum_i \frac{\partial f}{\partial q_i} \cdot dq_i$$

Where dq_i - error in dimension i

$\frac{\partial f}{\partial q_i}$ - Weighting functions

Non-correlated errors $$df = \sqrt{\sum_i \left(\frac{\partial f}{\partial q_i} \right)^2 \cdot q_i^2}$$

RDDS1 [MHz/μm]: $\frac{\partial f}{\partial b} \approx -1.2$ $\frac{\partial f}{\partial a} \approx 0.4 \div 0.6$ $\frac{\partial f}{\partial t} \approx -0.1$

For H90VG5 and H60VG3 $\frac{\partial f}{\partial q} \approx 1.2$ (10%-accuracy)

For estimation tolerances we suppose weight function independent of cell number or group velocity

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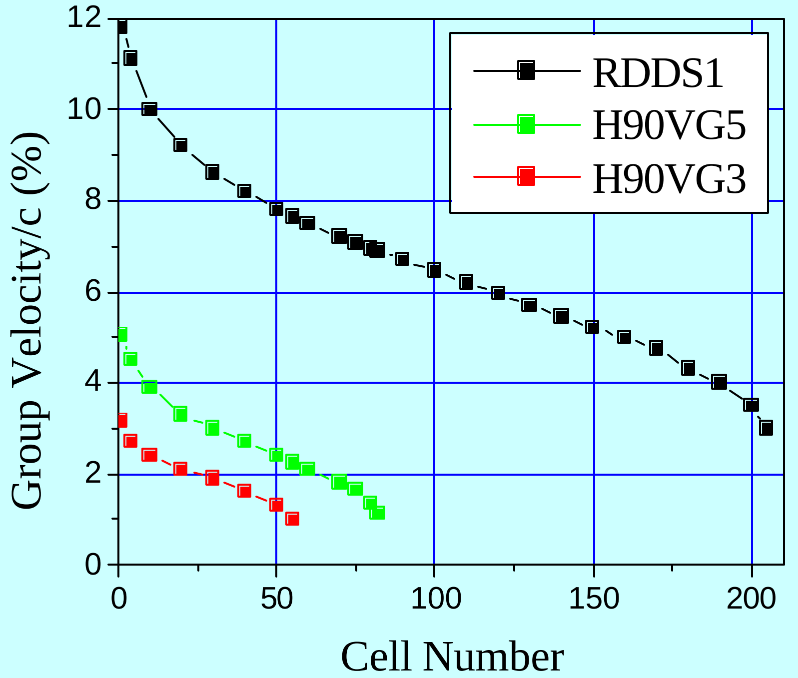
Group velocity profile in NLC structure.



Group velocity profile in RDDS1, H90VG5 and H60VG3 structures

Linear approximation $$b_g = \bar{b}_g \cdot (1 + 2a\,(1/2 - x))$$

where: $a = (ß_{max} - ß_{min})/2\bar{ß}_g$ x=z/L

Examples for NLC structure

•**RDDS1** is 1.8-m long traveling wave accelerating structure (*2p/3-mode)* made of 206 cells with dimensions tailored from cell to cell to detune dipole modes frequencies and provide the required properties of accelerating mode. RDDS1 has high group velocity (b_g=12% →2.7%).

•**H90VG5** is 0.9 m long traveling wave accelerating structure with a 150° phase advance (*5p/6-mode)* and 5% initial group velocity (b_g=5.06% →1.14%). Structure made of 83 cells.

•**H90VG3** is 0.9 m long traveling wave accelerating structure with a 150° phase advance (*5p/6-mode)* and 3% initial group velocity (b_g=3.2% →2.1%). Structure made of 83 cells.

•**H60VG3** is 0.6 m long traveling wave accelerating structure with a 150° phase advance (*5p/6-mode)* and 3% initial group velocity (b_g=3.16% →1.05%). Structure made of 55 cells and more optimized for 3% group velocity than H90VG3 structure.

* Z.Li Snowmass July 2001

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Random errors.

Lets df is randomly distributed along the structure. For homogeneous structure made of N cells with a constant group velocity along the structure.

$$\frac{\Delta E}{E} = \frac{(dq)_{rms}^2}{4} \cdot N$$ →gaussian

If errors distributed uniformly in range $\pm\delta\theta_{max}$, then $(dq)^2_{rms}=1/3((dq)^2_{max}$.and energy losses are:

$$\frac{\Delta E}{E} = \frac{(dq)_{\max}^2}{12} \cdot N$$ →uniform

For tapered structure with group velocity linear along the structure, summarizing contributions from all cells in LC we have

$$\frac{\Delta E}{E} = \frac{(\Delta E/E)_{av}}{(1-a^2)}$$

$(DE/E)_{a-}$ defined above for average group velocity

Frequency tolerance

$$df = \frac{2\bar{b}_g f_0}{\Theta_0} \cdot \sqrt{\frac{(\Delta E/E)\cdot(1-a^2)}{N}}$$

Tolerances in cell diameter:

$$d(2b) = \frac{2 \cdot df}{(\partial f/\partial b)}$$

The same frequency errors in cells at the end of the structure more affects to energy loss, than cells in the beginning of structure.

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Table for random frequency errors

Criteria - 1% energy loss

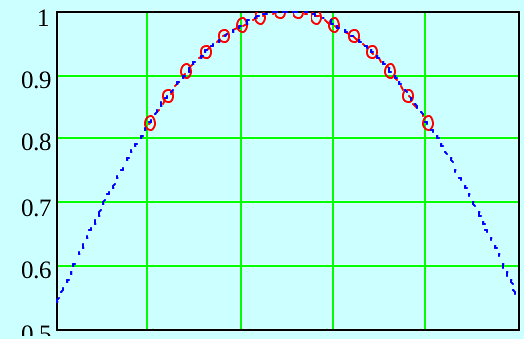
What tolerances for cell dimensions (frequencies) should be to satisfy that criteria?

Tolerances for random errors in assumption 1% energy loss (f_0=11.424GHz):

Structure Type	Θ_0	β %	$\alpha = \Delta\beta/\beta$	Normal Distribution (R.M.S.error) *d f* MHz	Normal Distribution (R.M.S.error) Δ(2b) µm	Uniform Distribution (Max error0 *d f* MHz	Uniform Distribution (Max error0 Δ(2b) µm
RDDS1	2π/3	6.4	0.47	4.2	7.0	7.3	12.0
H90VG5	5π/6	2.7	0.45	3.07	5.1	5.3	8. 8
H90VG3	5π/6	2.6	0.16	2.96	4.9	5.1	8.5
H60VG3	5π/6	2.0	0.175	2.87	4.8	5.0	8.3

Systematic frequency errors

Accelerating structure with constant group velocity



If structure made of cells having systematic frequency errors, the beam will continuously slip over wave. For identical cells with systematic error dq

$$\left(\frac{\Delta E}{E}\right)_s = 1-\cos(j_0)\cdot\frac{\sin(Nd\,q/2)}{Nd\,q/2}$$

j_{0}- -is klystron phase. Energy loss reaches minimum value when $\varphi_0=0$

$$\left(\frac{\Delta E}{E}\right)_s = N^2\cdot\frac{(dq)^2}{24}$$ →gaussian distribution

Formula is fair when all structures have the same systematic error dq (systematic-systematic errors) or when systematic errors from structure-to-structure are distributed normally ($dq=\sigma$). If systematic error uniformly distributed from structure to structure with the maximum error dq_{max},,

$$\left(\frac{\Delta E}{E}\right)_{syst} = N^2\cdot\frac{(dq_s)_{\max}^{\ 2}}{72}$$ →uniform distribution

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Systematic frequency errors for tapered structure

Tapered Accelerating structure.

Let's suppose: $b_g = \overline{b}_g \cdot (1 + a - 2a \cdot x)$ x=z/L $\in$ [0,1]– coordinate along AS.

Phase between beam and wave is equal: $q(x) = \dfrac{\overline{dq} \cdot N}{2 \cdot a} \cdot \ln\left(\dfrac{1+a}{1+a-2ax}\right) - f_0$

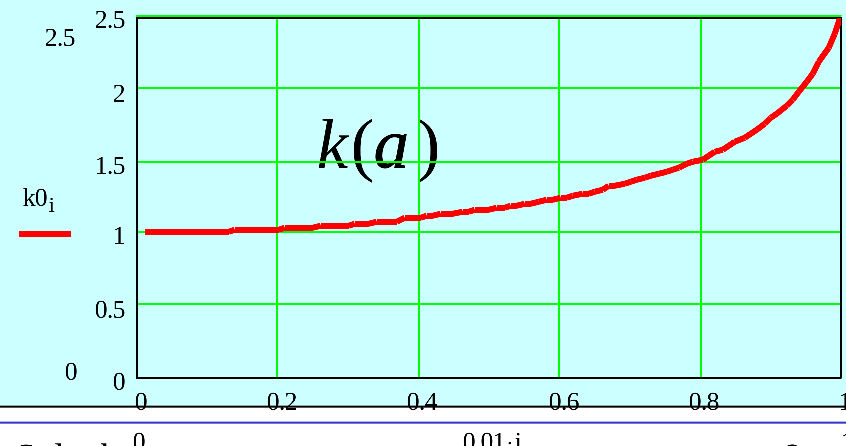
where: $\overline{dq} = \Theta_0 \cdot \dfrac{df}{\overline{b}_g \cdot f_0}$ - average phase error, f_0 - klystron phase

Energy loss is minimum for the klystron phase → $f_0 = \dfrac{\overline{dq} \cdot N}{2a} \cdot \left(1 - \dfrac{1-a}{2a} \cdot \ln\left(\dfrac{1+a}{1-a}\right)\right)$

Integrating along structure we have

$$\left(\frac{\Delta E}{E}\right)_s = N^2 \cdot \frac{(d\overline{q})^2}{24} \cdot k(a)$$

(for systematic errors randomly distributed from structure-to-structure, normal distribution)



Temperature compensation of systematic errors

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Systematic frequency errors can be partly compensated by changing the structure temperature. The frequency shift vs. structure temperature ($T=T_0+dT$) is defined by equation:

$$\frac{df}{f_0} = a_T \cdot dT$$

where: $=1.65\cdot 10^{-5}$ – cooper thermal expansion coefficient. At the range of temperature changing $dT=\pm 10°C$ we can compensate systematic error in frequency ***df=±1.84 MHz (D(2b)=3mm).*** The temperature control of the individual structure helps loose tolerances for systematic errors.

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Matching cells to reduce systematic errors

Effect of systematic errors can be reduced, if few cells are tunable. By tuning we produce additional phase jump in opposite direction to minimize energy loss. The maximum effect will be when phase jump is $Df = -(dq_{s}N)/2$. It allows reduce the energy loss by additional factor of 4.

$$\left(\frac{\Delta E}{E}\right)_{match} = N^2 \cdot \frac{(dq_s)_{rms}^{\;2}}{216}$$

Problem - power reflection from this section that increase surface electric field in structure.

$$\frac{\partial \Gamma}{\partial f} = \frac{q \cdot \cos(q)}{2 \cdot \sin(q)} \cdot \frac{1}{b_g \cdot f_0} \qquad [1/\mathrm{MHz}]$$

Reflection from enter and exit of this section will cancel each other if total phase advance in section is $n_{s*}\Theta = m\pi$, where n_s-number of cells in section, θ-phase advance in each cell. For example, matching section of 3 cells for 2π/3-mode or 6 cells for 5π/6-mode has no reflection. But even in this case we still will have reflection wave inside section.

To have required phase shift Df in n_s cells we should change frequency each cell to

$$\Delta f = \frac{\Delta f}{n_s} \cdot \frac{b_g \cdot f}{q}$$

Maximum reflection inside matching section

$$\Delta \Gamma = \frac{\cos(q)}{2\sin(q)} \cdot \frac{\Delta f}{n_s}$$

Table for systematic frequency errors

Tolerances for systematic frequency errors in assumption 1% energy loss (f=11.424GHz)

Structure Type	Θ_0	β %	$\alpha = \Delta\beta/\beta$	k	df MHz	$\Delta(2b)$ μm	df_{ma} MHz	$\Delta(2b)$ μm	df MHz	$\Delta(2b)$ μm	Δf
RDDS1	$2\pi/3$	6.4	0.47	1.1	0.79	1.32	1.37	2.30	2.74	4.60	
H90VG5	$5\pi/6$	2.7	0.45	1.1	0.72	1.20	1.25	2.08	2.50	4.16	-16
H60VG3	$5\pi/6$	2.0	0.175	1.04	0.80	1.33	1.38	2.32	2.76	4.64	-10

Gaussian cell | Uniform | Matching Compensation

Tolerance doesn't depend structure length

For optimized structures filling time $t = \frac{1}{4}\cdot\left(\frac{\Theta\cdot N}{f\cdot b_g}\right)\approx 100ns$ $\rightarrow$ $\frac{\Delta E}{E}\propto(df\cdot t)^2$

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Conclusion

Criteria 1% energy loss

- Tolerances for random errors in NLC structure (2b) are about 5μm RMS (±8.5 μm max) or 3 MHz RMS (± 5 MHz max) frequency error.
- Systematic errors are dominant. Tolerances in cell dimension are 1.3 μm RMS (±2.3μm max) or 0.8 MHz RMS (± 1.4 MHz max) frequency error.
- Tolerance to systematic errors will loose if use temperature control individually for each structure. ±10°C allow to loose tolerance to 4.2-4.3 μm RMS (the same level as for random errors).
- Matching section (3-6 cells depending of phase advance) also allow to loose tolerances for systematic errors to level 4.5 μm . Reflection inside matching section increase surface field up-to 10%. Possible problems – changing frequency distribution of HOM.